2024 UNIST Workshop on the Introduction to the Theory of Elliptic PDEs

Date October 23-25, 2024

Place Bldg. 108, Room 320, UNIST, Ulsan, Korea

Organized by Youngae Lee (UNIST)

Supported by

Schedule

	10.23 (Wed.)	10.24 (Thur.)	10.25 (Fri.)
-10:00	Breakfast	Breakfast	Breakfast
10:00-11:30	Jongmin Han (Kyung Hee University)	Namkwon Kim (Chosun University)	Francesca De Marchis (Università La Sapienza)
11:30-13:30	Lunch	Lunch	Lunch
13:30-14:30	Free discussion	Free discussion	Free discussion
14:30-16:00	Ting-Jung Kuo (National Taiwan Normal University)	Zhijie Chen (Tsinghua University)	Michiaki Onodera (Tokyo Institute of Technology)
16:00-16:30	Coffee Break	Coffee Break	Coffee Break
16:30-18:00	Weiwei Ao (Wuhan University)	Aleks Jevnikar (University of Udine)	Xinyu Li (Tsinghua University)
18:00-	Dinner	Dinner	Dinner

Jongmin Han (Kyung Hee University)
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Title: Solution structures for the self-dual equations of the Einstein-Maxwell-Higgs model on compact surfaces

Abstract: In this talk, we consider the self-dual equations arising from the Einstein-Maxwell-Higgs model on compact surfaces. According to the behaviors of solutions as the coupling parameter ε decreases to zero, we classify solutions into two categories: topological solutions and nontopological solutions. In this talk, we review recent advances for the exsitence of solutions for both types. First, we show that there exists ε_0 such that there are at least two solutions $v_{1,\varepsilon}$ and $v_{2,\varepsilon}$ for $\varepsilon IN(0, \varepsilon_0)$ and no solutions for $\varepsilon > \varepsilon_0$. It turns out that $v_{1,\varepsilon}$ is topological for small ε. Second, we present the existence of nontopological solutions for all small ε via a degree argument.

Ting-Jung Kuo (National Taiwan Normal University)
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Title: Spherical metric with conical singularties with total curvature on flat tori

Abstract: Let E_τ , τINH be a flat torus with periods 1 and τ, and set $\omega_0 = 0,\ \omega_1 = 1,\ \omega_2 = \tau,\ \omega_3 = 1+\tau$. In this talk, we investigate the following curvature equation:

$$\Delta u + e^{u} = \sum_{k=0}^{3} 4\pi\, m_k \delta_{\frac{\omega_k}{2}} \quad E_\tau \qquad (1),$$

where $m_k INZ$ and $\sum_{k=0}^{3} 4\pi m_k$ is called the total curvature. This equation (1) arised originally from conformal geometry and any solution to this equation produces a conformal metric $g = \frac{1}{2} e^{u} ds^2$ on the torus E_τ, where ds^2 denotes the flat metric on E_τ. The new metric exhibits a spherical metric but has conical singularities on each $\frac{\omega_k}{2}$ with angle $2\pi\,(1+m_k)$ When $\sum_{k=0}^{3} m_k$ is even, from PDE point of view, the bubbling phenomenon might happen, which makes the equation (1) become extremely difficult to study. In this talk, we will explain how to study the case $\sum_{k=0}^{3} m_k$ is even by the method of monodromy theory.

Weiwei Ao (Wuhan University)
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Title: On the vortex solution for the SQG equation and Euler equation

Abstract: In this talk, I will talk about the vortex solution for the Euler equation. Firstly, we consider the generalization of the 2D Euler equation, i.e. the generalized surface quasi-geostrophic equation, we construct travelling and rotating solutions such that the active scalar has compact support. A key element in the construction is the non-degeneracy for the solutions of the fractional plasma equation. I will also talk about the 3D Euler equation, for which we construct travelling solutions of multiple vortex rings. This is based on joint work with Davila, Liu, Del Pino, Musso, Wei.

Namkwon Kim (Chosun University)
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Title: Bubbling nontopological solutions for two by two systems

Abstract: We present some of recent progress on the existence and uniqueness of radial nontopological solutions of certain two by two Chern-Simons systems. It turns out that he coefficient of the nonlinear interaction terms involves seriously on the number of crossing points and hence the degree of the system varies drastically as the coefficient crosses certain critical values.

Zhijie Chen (Tsinghua University)
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Title: Singular SU(3) Toda systems on torus

Abstract: In this talk, I will survey our main results from a series of papers about singular SU(3) Toda systems on torus. For the non-critical case, we give a sharp upper bound of the number of solutions. For the critical case, we give the structure of even solutions.

Aleks Jevnikar (University of Udine)
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Title: On the bifurcation diagram for a free boundary plasma problem

Abstract: We are concerned with qualitative properties of the bifurcation diagram of a problem arising in plasma physics, showing in particular uniqueness and monotonicity of its solutions. Generic properties of the global bifurcation diagram are also discussed. Joint project with D. Bartolucci (Roma), Y. Hu (Changsha), W. Yang (Wuhan).

Francesca De Marchis (Universita La Sapienza)
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Title: Asymptotic behavior of solutions of problems involving power nonlinearities

Abstract: We discuss, highlighting analogies and differences, the asymptotic behavior both of solutions to the classical Lane Emden equation both to a linear elliptic equation subject to a Neumann boundary condition involving a power nonlinearity when the power tends to a critical value. Based on joint papers with M. Grossi, H. Fourti, I. Ianni and F. Pacella.

Michiaki Onodera (Tokyo Institute of Technology)
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Title: A perturbation theory of overdetermined boundary value problems

Abstract: Our main interest lies in the shape of a bounded domain for which a parametrized overdetermined boundary value problem admits a solution. Unlike a typical nonlinear problem where the non-degeneracy of the linearized operator implies a local one-to-one correspondence between parameters and solutions, overdetermined problems generally fail to follow this scenario because of a loss of derivatives. In this talk, I will explain a general perturbation result in overdetermined problems based on a characterization of an evolving domain by a geometric evolution equation. It turns out that the non-degeneracy and an additional monotonicity condition of the linearized operator are the properties inherited by the original problem, i.e., these linear properties imply the existence of a monotonically increasing family of domains admitting solvability of the corresponding overdetermined problem under a small continuous deformation of parameters.

Xinyu Li (Tsinghua University)
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Title: Uniqueness of Ground states for Fractional Schrödinger Equations with General Nonlinearities

Abstract: In 2013, R.L. Frank and E. Lenzmann study the following problem:

$$(-\Delta)^s u + u = u^{p-1} \text{in } R^N, \tag{1}$$

where $s \in (0,1)$, $N = 1$, $p \in (2, 2^*_s)$ and 2^*_s is the critical fractional Sobolev exponent. They proved that the ground state is unique (up to translations). Then in 2016, they, together with L. Silvestre showed similar uniqueness results for high dimensions ($N \geq 2$), in which they proposed a challenging open problem to extend their results about non-degeneracy and uniqueness of ground states to nonlinearities $f(u)$ beyond the pure-power case. In this talk, we use a continuation argument to prove the uniqueness of ground states for Fractional Schrödinger Equations with a large class of convex nonlinearities, under the assumption that the positive solution u of (1) is non-degenerate.